



Serie 2, Musterlösung

Datum: HS 24

1. Wurzeln

53XZ9V

Vereinfachen Sie so weit wie möglich

(a) $\sqrt{50}$

(c) $\sqrt{x^2}$

(b) $\sqrt{\frac{14}{81}}$

(d) $\sqrt{25c^6e^4}$

(e) $\sqrt{16 + 25}$

Lösung:

(a) $\sqrt{50}$

$$= \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

(b) $\sqrt{\frac{14}{81}}$

$$= \frac{\sqrt{14}}{\sqrt{81}} = \frac{\sqrt{14}}{9}$$

(c) $\sqrt{x^2}$

$$= |x|$$

(d) $\sqrt{25c^6e^4}$

$$= \sqrt{25} \cdot \sqrt{c^6} \cdot \sqrt{e^4} = 5c^3e^2$$

(e) $\sqrt{16 + 25}$

$$= \sqrt{41}$$

2. Vereinfachen

7RPQJI

(a) $\sqrt{h^2} \cdot \sqrt{h^2}$

(e) $4\sqrt{1000}$

(b) $\frac{\sqrt{45}}{\sqrt{5}}$

(f) $\frac{\sqrt{27}}{\sqrt{6}}$

(c) $\sqrt{18} \cdot \sqrt{10}$

(g) $\sqrt{5} \cdot (3 + 4\sqrt{5})$

(d) $\sqrt{98}$

(h) $(2 + \sqrt{6}) \cdot (2 - \sqrt{6})$

Lösung:

(a)

$$\sqrt{h^2} \cdot \sqrt{h^2} = |h| \cdot |h| = h^2$$

(b)

$$\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3$$

(c)

$$\begin{aligned} & \sqrt{18} \cdot \sqrt{10} \\ &= \sqrt{18 \cdot 10} = \sqrt{180} = \sqrt{36 \cdot 5} = 6\sqrt{5} \end{aligned}$$

(d)

$$\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$$

(e)

$$4\sqrt{1000} = 4\sqrt{100 \cdot 10} = 4 \cdot 10 \cdot \sqrt{10} = 40\sqrt{10}$$

(f)

$$\frac{\sqrt{27}}{\sqrt{6}} = \sqrt{\frac{27}{6}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

(g)

$$\begin{aligned} & \sqrt{5} \cdot (3 + 4\sqrt{5}) \\ &= 3\sqrt{5} + 4\sqrt{5} \cdot \sqrt{5} = 3\sqrt{5} + 4 \cdot 5 = 3\sqrt{5} + 20 \end{aligned}$$

(h)

$$(2 + \sqrt{6}) \cdot (2 - \sqrt{6}) = 2^2 - (\sqrt{6})^2 = 4 - 6 = -2$$

3. Gebrochene Exponenten

9K6A7M

Vereinfachen Sie ohne Taschenrechner

(a) $16^{1/4}$

(d) $8^{4/3}$

(b) $9^{3/2}$

(e) $32^{3/5}$

(c) $64^{2/3}$

(f) $(\sqrt{2})^6$

Lösung:

(a) $16^{1/4}$

$$= (2^4)^{1/4} = 2^{4 \cdot \frac{1}{4}} = 2^1 = 2$$

(b) $9^{3/2}$

$$= (3^2)^{3/2} = 3^{2 \cdot \frac{3}{2}} = 3^3 = 27$$

(c) $64^{2/3}$

$$= (4^3)^{2/3} = 4^{3 \cdot \frac{2}{3}} = 4^2 = 16$$

(d) $8^{4/3}$

$$= (2^3)^{4/3} = 2^{3 \cdot \frac{4}{3}} = 2^4 = 16$$

(e) $32^{3/5}$

$$= (2^5)^{3/5} = 2^{5 \cdot \frac{3}{5}} = 2^3 = 8$$

(f) $(\sqrt{2})^6$

$$= (2^{1/2})^6 = 2^{1/2 \cdot 6} = 2^3 = 8$$

4. Negative Exponenten

N5CIQK

Vereinfachen Sie bis nur ein positiver Exponent übrig bleibt.

(a) $(-3)^{-2}$

(c) $\frac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$

(b) 13^0

(d) $(-\frac{3}{4})^{-3}$

Lösung:

(a)

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

(b)

$$13^0 = 1$$

(c)

$$\frac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}} = \frac{4 \cdot \frac{1}{3^2}}{\frac{1}{2^2} \cdot \frac{1}{3}} = \frac{4}{9} \cdot \frac{4 \cdot 3}{1} = \frac{4 \cdot 4 \cdot 3}{9} = \frac{48}{9} = \frac{16}{3}$$

(d)

$$\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3 = -\frac{4^3}{3^3} = -\frac{64}{27}$$

5. Exponenten gemischt

GTGFQV

Vereinfachen Sie, so dass nur positive Exponente übrig bleiben.

(a) $(-xy^3)^2$

(e) $\frac{(x+4y)^{1/2}}{2(x+4y)^{-1}}$

(b) $(2x^3y^{-5}) \cdot (2x^{-1}y^3)^4$

(f) $\frac{x^{1/3} + x^{1/4}}{x^{1/2}}$

(c) $(-32)^{3/5}$

(d) $\frac{4a^3b^5}{(2a^2b)^4} \cdot \frac{b^{-1}}{a^{-3}}$

(g) $\sqrt[3]{24x^6y^{12}}$

Lösung:

(a)

$$(-xy^3)^2 = (-1)^2 \cdot x^2 \cdot (y^3)^2 = x^2 \cdot y^6$$

(b)

$$(2x^3y^{-5}) \cdot (2x^{-1}y^3)^4$$

$$= 2x^3y^{-5} \cdot 16x^{-4}y^{12} = 32x^{3-4}y^{-5+12} = 32x^{-1}y^7 = \frac{32y^7}{x}$$

(c)

$$(-32)^{3/5} = (-(2^5))^{3/5} = -2^{5 \cdot \frac{3}{5}} = -2^3 = -8$$

(d)

$$\frac{4a^3b^5}{(2a^2b)^4} \cdot \frac{b^{-1}}{a^{-3}}$$

$$= \frac{4a^3b^5}{16a^8b^4} \cdot \frac{b^{-1}}{a^{-3}} = \frac{4a^3b^5 \cdot b^{-1} \cdot a^3}{16a^8b^4} = \frac{4a^{3+3}b^{5-1}}{16a^8b^4}$$

$$= \frac{4a^6b^4}{16a^8b^4} = \frac{a^6}{4a^8} = \frac{1}{4a^2} = \frac{1}{4a^2}$$

(e)

$$\frac{(x+4y)^{1/2}}{2(x+4y)^{-1}}$$

$$= \frac{(x+4y)^{1/2}}{2 \cdot \frac{1}{(x+4y)}} = \frac{(x+4y)^{1/2} \cdot (x+4y)}{2} = \frac{(x+4y)^{3/2}}{2}$$

(f)

$$\frac{x^{1/3} + x^{1/4}}{x^{1/2}}$$

$$= \frac{x^{1/3}}{x^{1/2}} + \frac{x^{1/4}}{x^{1/2}} = x^{1/3-1/2} + x^{1/4-1/2} = x^{-1/6} + x^{-1/4}$$

$$= \frac{1}{x^{1/6}} + \frac{1}{x^{1/4}}$$

(g)

$$\sqrt[3]{24x^6y^{12}}$$

$$= (24x^6y^{12})^{1/3} = 24^{1/3} \cdot (x^6)^{1/3} \cdot (y^{12})^{1/3} = 2 \cdot x^{6/3} \cdot y^{12/3}$$

$$= 2x^2y^4 = 2x^2y^4$$