Serie 11, Basiswechsel 2D

Klasse: 1Ea, 1Eb, 1Sb — Datum: HS 17

1. Orthonormalbasis — UFDF5I

Geben Sie die Punkte in der jeweiligen Orthonormalbasis an.

(a) Basis $\vec{f}_1 = \begin{pmatrix} 3/5 \\ -(4/5) \end{pmatrix}, \vec{f}_2 = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$.

$$\vec{A} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}, \vec{B} = \begin{pmatrix} -2.2 \\ -0.4 \end{pmatrix}, \vec{C} = \begin{pmatrix} 6.8 \\ -7.4 \end{pmatrix}, \vec{D} = \begin{pmatrix} 10.4 \\ -2.2 \end{pmatrix}$$

(b) Basis $\vec{f}_1 = \begin{pmatrix} 8/17 \\ 15/17 \end{pmatrix}, \vec{f}_2 = \begin{pmatrix} 15/17 \\ -(8/17) \end{pmatrix}$.

$$\vec{A} = \begin{pmatrix} -0.941176 \\ -1.76471 \end{pmatrix}, \vec{B} = \begin{pmatrix} 3.35294 \\ 8.41176 \end{pmatrix}, \vec{C} = \begin{pmatrix} 6.29412 \\ 1.17647 \end{pmatrix}, \vec{D} = \begin{pmatrix} -2.70588 \\ -0.823529 \end{pmatrix}$$

(c) Basis $\vec{f}_1 = \begin{pmatrix} -(5/13) \\ 12/13 \end{pmatrix}, \vec{f}_2 = \begin{pmatrix} 12/13 \\ 5/13 \end{pmatrix}$.

$$\vec{A} = \begin{pmatrix} -4.154 \\ 4.769 \end{pmatrix}, \vec{B} = \begin{pmatrix} -4.538 \\ 5.692 \end{pmatrix}, \vec{C} = \begin{pmatrix} -1.769 \\ 6.846 \end{pmatrix}, \vec{D} = \begin{pmatrix} 6.462 \\ 2.692 \end{pmatrix}$$

(d) Basis $\vec{f}_1 = \begin{pmatrix} 7/25 \\ -(24/25) \end{pmatrix}, \vec{f}_2 = \begin{pmatrix} 24/25 \\ 7/25 \end{pmatrix}$.

$$\vec{A} = \begin{pmatrix} 6.84 \\ 5.12 \end{pmatrix}, \vec{B} = \begin{pmatrix} -0.92 \\ -7.56 \end{pmatrix}, \vec{C} = \begin{pmatrix} 0.68 \\ 1.24 \end{pmatrix}, \vec{D} = \begin{pmatrix} 7.28 \\ 0.04 \end{pmatrix}$$

2. Orthogonalbasis — V6P3LC

Geben Sie die Punkte in der jeweiligen Orthongonalbasis an.

(a) Lösung durch Projektion:

$$\begin{aligned} A_1^F \cdot \vec{f}_1 &= \vec{A} \odot \frac{\vec{f}_1}{\left|\vec{f}_1\right|} \cdot \frac{\vec{f}_1}{\left|\vec{f}_1\right|} = \frac{\left(\vec{A} \odot \vec{f}_1\right)}{\left|\vec{f}_1\right|^2} \cdot \vec{f}_1 \\ &= \frac{\begin{pmatrix} 2 \\ -11 \end{pmatrix} \odot \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{5^2} \cdot \vec{f}_1 = \frac{6+44}{25} \cdot \vec{f}_1 = 2 \cdot \vec{f}_1 \end{aligned}$$

$$\begin{aligned} A_2^F \cdot \vec{f_2} &= \frac{\left(\vec{A} \odot \vec{f_2}\right)}{\left|\vec{f_2}\right|^2} \cdot \vec{f_2} \\ &= \frac{\begin{pmatrix} 2 \\ -11 \end{pmatrix} \odot \begin{pmatrix} 4 \\ 3 \end{pmatrix}}{5^2} \cdot \vec{f_2} = \frac{8-33}{25} \cdot \vec{f_1} = -1 \cdot \vec{f_2} \end{aligned}$$

Also $\vec{A} = 2 \cdot \vec{f_1} - 1 \cdot \vec{f_2}$ oder $\vec{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}_F$.

$$\begin{aligned} B_1^F \cdot \vec{f_1} &= \frac{\begin{pmatrix} 5 \\ -15 \end{pmatrix} \odot \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{5^2} \cdot \vec{f_1} = \frac{15+60}{5^2} \cdot \vec{f_1} = 3 \cdot \vec{f_1} \\ B_2^F \cdot \vec{f_2} &= \frac{\begin{pmatrix} 5 \\ -15 \end{pmatrix} \odot \begin{pmatrix} 4 \\ 3 \end{pmatrix}}{5^2} \cdot \vec{f_1} = \frac{20-45}{5^2} \cdot \vec{f_1} = -1 \cdot \vec{f_2} \end{aligned}$$

Also $\vec{B} = 3 \cdot \vec{f_1} - 1 \cdot \vec{f_2}$ oder $\vec{B} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}_F$.

$$\begin{aligned} C_1^F &= \frac{\begin{pmatrix} -2 \\ 11 \end{pmatrix} \odot \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{5^2} = \frac{-6-44}{5^2} = -2 \\ C_2^F &= \frac{\begin{pmatrix} -2 \\ 11 \end{pmatrix} \odot \begin{pmatrix} 4 \\ 3 \end{pmatrix}}{5^2} = \frac{-8+33}{5^2} = 1 \end{aligned}$$

Also $\vec{C} = -2 \cdot \vec{f_1} + 1 \cdot \vec{f_2}$ oder $\vec{C} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}_F$.
Alternativ mit der Basismatrix:

$$\left[\begin{pmatrix} 3 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}\right] \odot \begin{pmatrix} D_1^F \\ D_2^F \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$$

Die Basis-Matrix ist zwar nicht orthogonal, aber es gilt $\mathbf{F}^\intercal \odot \mathbf{F} = \mathbf{K}$, wobei $\mathbf{K}$ eine Diagonalmatrix ist. Wir berechnen also

$$\underbrace{\mathbf{F}^\intercal \odot \mathbf{F}}_{=\begin{pmatrix}25 & 0\\ 0 & 25\end{pmatrix}} \begin{pmatrix}D_1^F\\ D_2^F\end{pmatrix} = \mathbf{F}^\intercal \odot \begin{pmatrix}10\\ -5\end{pmatrix}$$

$$\begin{pmatrix}25 & 0\\ 0 & 25\end{pmatrix} \odot \begin{pmatrix}D_1^F\\ D_2^F\end{pmatrix} = \begin{pmatrix}50\\ 25\end{pmatrix}$$

$$\begin{pmatrix}D_1^F\\ D_2^F\end{pmatrix} = \begin{pmatrix}50/25\\ 25/25\end{pmatrix} = \begin{pmatrix}2\\ 1\end{pmatrix}$$

(b) Basis $\vec{f}_1 = \begin{pmatrix}2\\ 1\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}1\\ -2\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}15\\ 0\end{pmatrix}, \vec{B} = \begin{pmatrix}-3\\ 6\end{pmatrix}, \vec{C} = \begin{pmatrix}5\\ -15\end{pmatrix}, \vec{D} = \begin{pmatrix}12\\ -14\end{pmatrix}$$

(c) Basis $\vec{f}_1 = \begin{pmatrix}5\\ 1\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}-1\\ 5\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}-7\\ 9\end{pmatrix}, \vec{B} = \begin{pmatrix}11\\ -3\end{pmatrix}, \vec{C} = \begin{pmatrix}-5\\ 25\end{pmatrix}, \vec{D} = \begin{pmatrix}3\\ 11\end{pmatrix}$$

(d) Basis $\vec{f}_1 = \begin{pmatrix}3\\ -2\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}2\\ 3\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}-4\\ 7\end{pmatrix}, \vec{B} = \begin{pmatrix}1\\ -5\end{pmatrix}, \vec{C} = \begin{pmatrix}8\\ -1\end{pmatrix}, \vec{D} = \begin{pmatrix}6\\ 9\end{pmatrix}$$

3. Allgemeine Basis — AKD6C2

(a) Basis $\vec{f}_1 = \begin{pmatrix}1\\ 0\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}4/5\\ 3/5\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}1.2\\ -0.6\end{pmatrix}, \vec{B} = \begin{pmatrix}2.2\\ -0.6\end{pmatrix}, \vec{C} = \begin{pmatrix}-1.2\\ 0.6\end{pmatrix}, \vec{D} = \begin{pmatrix}2.8\\ 0.6\end{pmatrix}$$

(b) Basis $\vec{f}_1 = \begin{pmatrix}1\\ 0\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}-1\\ 3\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}3\\ 9\end{pmatrix}, \vec{B} = \begin{pmatrix}3\\ -9\end{pmatrix}, \vec{C} = \begin{pmatrix}-8\\ 21\end{pmatrix}, \vec{D} = \begin{pmatrix}-6\\ 24\end{pmatrix}$$

(c) Basis $\vec{f}_1 = \begin{pmatrix}5\\ 1\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}-5\\ 1\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}-15\\ 1\end{pmatrix}, \vec{B} = \begin{pmatrix}15\\ 1\end{pmatrix}, \vec{C} = \begin{pmatrix}-25\\ 5\end{pmatrix}, \vec{D} = \begin{pmatrix}-5\\ 3\end{pmatrix}$$

(d) Basis $\vec{f}_1 = \begin{pmatrix}1\\ -2\end{pmatrix}, \vec{f}_2 = \begin{pmatrix}0\\ 1\end{pmatrix}$.

$$\vec{A} = \begin{pmatrix}-2\\ 5\end{pmatrix}, \vec{B} = \begin{pmatrix}1\\ -3\end{pmatrix}, \vec{C} = \begin{pmatrix}2\\ -3\end{pmatrix}, \vec{D} = \begin{pmatrix}0\\ 3\end{pmatrix}$$